MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL TECHNICAL UNIVERSITY
«DNIPRO POLYTECHNIC»

PROBLEMS OF KINEMATICS

COMPILED
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P r o b l e m. A train travelling at a velocity $v_0 = 54$ km/h stops in $t_1 = 2$ min after braking starts. Assuming the motion of the train during braking to be uniformly retarded, determine the distance covered during the braking time.

S o l u t i o n. The problem states that the motion is uniformly retarded:

$$x = v_0 t + a\frac{t^2}{2},$$

where x is measured from the place where braking began (therefore $x_0 = 0$). The velocity is

$$v = v_0 + at.$$

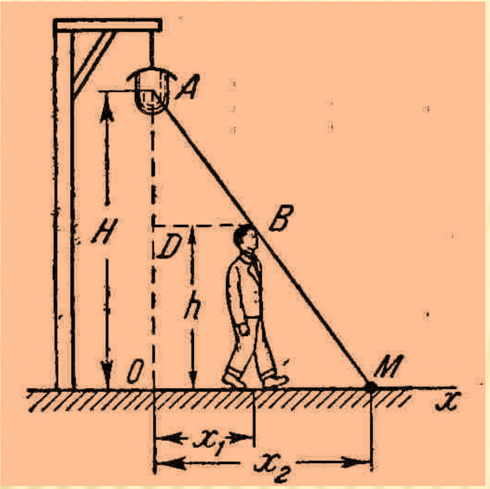
As the train stops at $t = t_1$, its velocity at that instant is $v_1 = 0$. Substituting these values in second equation, we obtain $0 = v_0 + at_1$, whence we find the acceleration:

$$a = -\frac{v_0}{t_1}.$$

Substituting the value of a into first equation and assuming $t = t_1$, we obtain the required distance:

$$x_1 = \frac{v_0 t_1}{2} = 900\ m.$$

P r o b l e m. A man of height h walks away from a lamp hanging at a height H with a velocity u. Determine the velocity of the tip of the man's shadow.



S o l u t i o n. First let us establish the law of motion of the tip of the shadow.

Depicting the man at an arbitrary distance x_1 from O, we find that the tip of his shadow is at x_2.

By virtue of the similarity of triangle OAM and DAB, we have: $x_2 = \frac{H}{H-h} x_1$.

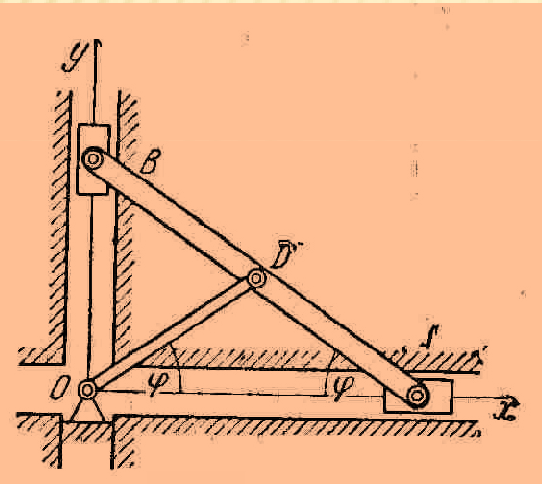
This is the equation of motion for the tip of the shadow M, provided the equation of motion for the man, i.e., $x_1 = f(t)$, is known.

Differentiating both parts of the equation with respect to time and noting that $\frac{dx_1}{dt} = u$ and $\frac{dx_2}{dt} = v$, where v is the required velocity, we obtain:

$$v = \frac{H}{H-h} u.$$

If the man moves with uniform velocity ($u = const$), the velocity of the shadow v is also uniform, but it is $\frac{H}{H-h}$ times faster than that of the man.

P r o b l e m. Blocks A and B of the mechanism are connected by a rod AB of length $l = 30$ cm and move in mutually perpendicular directions when the crank rotates. The crank OD of length $\frac{l}{2}$ is hinged to the middle of the rod AB to D. Develop the equations of motion for the sliding blocks A and B if angle φ increases in proportion to time (such rotation is called uniform) and the speed of rotation of the crank is 2 rpm. Determine the velocity and acceleration of the blocks at the instant when angle $\varphi = 30°$.



S o l u t i o n. According to the conditions of the problem, $\varphi = kt$, where k is a constant factor. We also know that, at $t = 60$ sec, angle $\varphi = 4\pi$ (two revolutions). Hence, $4\pi = 60k$, and $k = \pi/15$.

As $OD = AD$, $\angle OAB = \varphi$. Hence, $x_A = l\cos\varphi$, and $y_B = l\sin\varphi$, or $x_A = l\cos kt$; $y_B = l\sin kt$.

Differentiating x_A and y_B with respect to time, we obtain the velocity and acceleration of the blocks:

$v_A = -kl\sin kt$, $a_A = -k^2 l\cos kt$; $v_B = kl\cos kt$, $a_B = -k^2 l\sin kt$.

When angle $\varphi = 30°$, $kt = \frac{\pi}{6}$.

At that instant $v_A = -\frac{kl}{2} = -3.14 cm/sec$, $a_A = -\frac{k^2 l\sqrt{3}}{2} = -1.14\ cm/sec^2$,

$$v_B = \frac{kl\sqrt{3}}{2} = 5.44\frac{cm}{sec}, \quad a_B = -\frac{k^2 l}{2} = -0.66\ cm/sec^2.$$

P r o b l e m. The motion of a particle is described by the equations

$$x = 8t - 4t^2, \qquad y = 6t - 3t^2.$$

where x and y are in meters and t is in seconds. Determine the path, velocity and acceleration of the particle.

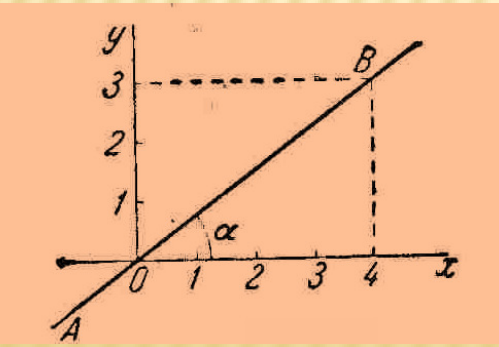
S o l u t i o n. To determine the path, we first eliminate time t from the equations of motion. Multiplying both parts of the first equation by 3 and both parts of the second by 4, and subtracting the second from the first, we obtain $3x - 4y = 0$, or $y = \frac{3}{4}x$.

Let us determine the velocity of the particle:

$$v_x = \frac{dx}{dt} = 8(1-t), \quad v_y = \frac{dy}{dt} = 6(1-t); \; v = \sqrt{v_x^2 + v_y^2} = 10(1-t).$$

Determine the acceleration of the particle,

$$a_x = \frac{d^2x}{dt^2} = -8, \qquad a_y = \frac{d^2y}{dt^2} = -6, \qquad a = 10\ m/sec^2.$$



Vectors v and a are evidently directed along the path, i.e., along AB. As $a = const.$, the motion is uniformly variable.

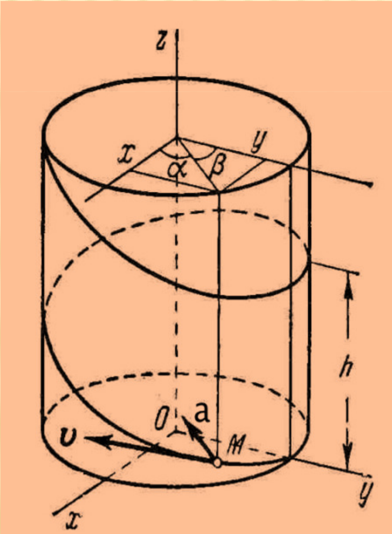
P r o b l e m. The motion of a particle is described by the equations:

$$x = l \sin \omega t, \qquad y = l \cos \omega t, \qquad z = ut,$$

where l, ω and u are constants. Determine the path, velocity and acceleration of the particle.

S o l u t i o n. Squaring the first two equations and adding them, we obtain (since $\sin^2 \omega t + \cos^2 \omega t = 1$): $x^2 + y^2 = l^2$.

Hence, the path lies on a circular cylinder of radius l, the axis of which is coincident with the z axis.



Determining t from the third equation and substituting its value into first, we find: $x = l \sin\left(\frac{\omega}{u} z\right)$.

Thus, the path of the particle is the line of intersection of a sinusoidal surface, whose generators are parallel to the y axis, with the cylindrical surface of radius l. This curve is called a *screw*.

It can be seen from the equations of motion that the particle makes one turn along the screw line in time t_1, determined by the equation $\omega t_1 = 2\pi$.

This displacement of the particle parallel to the z axis in that time is $h = ut_1 = \frac{2\pi u}{\omega}$ and is called the *pitch* of the screw.

Differentiating the equations of motion with respect to time, we obtain:

$$v_x = l\omega \cos \omega t\,, \qquad v_y = -l\omega \sin \omega t\,, \qquad v_z = u,$$

whence $v = \sqrt{l^2\omega^2(cos^2\omega t + sin^2\omega t) + u^2} = \sqrt{l^2\omega + u^2}$.

Calculate the projections of the acceleration:

$$a_x = -l\omega^2 \sin \omega t\,, \qquad a_y = -l\omega^2 \cos \omega t\,, \qquad a_z = 0,$$

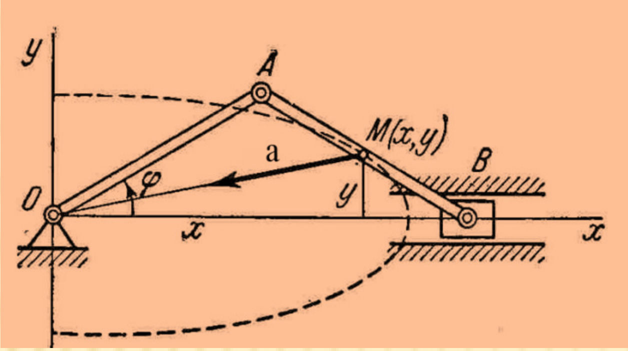
whence $$a = \sqrt{a_x^2 + a_y^2} = l\omega^2.$$

Thus, the motion has an acceleration of constant magnitude. To determine the direction of acceleration, we have the equations: $\cos \alpha_1 = \frac{a_x}{a} = -\sin \omega t = -\frac{x}{a}$, $\cos \beta_1 = \frac{a_y}{a} = -\cos \omega t = -\frac{y}{l}$, $\cos \gamma_1 = \frac{a_z}{a} = 0$.

Eevidently $\frac{x}{l} = \cos \alpha$, $\frac{y}{l} = \cos \beta$, where α and β are the angles made by the radius l, drawn from the axis of the cylinder to the moving particle, with the x and y axes.

As the cosines of angles α_1 and β_1 differ from the cosines of the angles α and β only in sign, we conclude that the acceleration of the particles is continuously directed along the radius of the cylinder towards its axis.

P r o b l e m. Determine the path, velocity and acceleration of point M in the middle of the connecting rod of the crank, if $OA = AB = 2r$ and angle φ increases in proportion with time: $\varphi = \omega t$.



S o l u t i o n. Let us develop the equations of motion of point M:

$$x = 2r\cos\varphi + r\cos\varphi\,, y = r\sin\varphi\,.$$

Substituting the expression for φ, we obtain the equations of motion of point M:

$$x = 3r\cos\omega t\,, \qquad y = r\sin\omega t\,.$$

To determine the path of M we write the equations of motion in the form:

$$\frac{x}{3r} = \cos\omega t\,, \qquad \frac{y}{r} = \sin\omega t\,.$$

Squaring these equations and adding them, we obtain: $\frac{x^2}{9r^2} = \frac{y^2}{r^2} = 1.$

Thus, the path described by point M is an ellipse with semiaxes equal $3a$ and a.

Determine the velocity of point M:

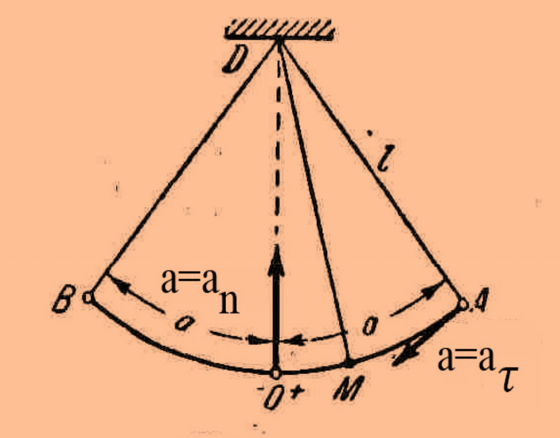
$$V_x = -3r\omega\sin\omega t\,, \qquad V_y = r\omega\cos\omega t\,; \quad V = r\omega\sqrt{9sin^2\omega t + cos^2\omega t}.$$

Now determine the acceleration of point M: $a_x = -3r\cos\omega t = -\omega^2 x,$

$a_y = -r\omega^2\sin\omega t = -\omega^2 y,$ whence $a = \sqrt{\omega^4(x^2 + y^2)} = \omega^2 r,$

where r is the radius vector from the origin to point M.

P r o b l e m. Small oscillations of the pendulum are represented by the equation of motion $s = l\sin kt$ (the origin is at O, l and k are constants). Determine the velocity, tangential and normal accelerations of the bob and the positions at which they become zero if the bob describes a circular arc of radius l.



S o l u t i o n. From the respective equations we find:

$$v = \frac{ds}{dt} = lk\cos kt\,; \quad a_\tau = \frac{dv}{dt} = -lk^2\sin kt,$$

$$a_n = \frac{v^2}{l} = \frac{l^2k^2}{l}\cos kt\,.$$

The equation of motion is that of simple harmonic motion, the amplitude of swing being l. In the extreme positions A and B, $\sin kt = \pm 1$, and consequently, $\sin kt = 0$ while $\cos kt = 1$. In this position, $a_\tau = 0$ and v and a_n have their maximum values:

$$v_{max} = lk, \qquad a_{n\,max} = \frac{l^2k^2}{l}.$$

P r o b l e m. A train starts moving from rest with uniform acceleration along a curve of radius R = 800 m and reaches a velocity $v_1 = 36$ km/h after travelling a distance $s_1 = 600$ m. Determine the velocity and acceleration of the train at the middle of this distance.

S o l u t i o n. As the train moves with uniform acceleration and $v_0 = 0$, its equation of motion (assuming $s_0 = 0$) is $s = \frac{1}{2} a_\tau t^2$, and velocity is $v = a_\tau t$.

Eliminating time t from these equations, we obtain $v^2 = 2a_\tau s$.

According to the conditions of the problem, at $s = s_1$, $v = v_1$, whence we find:

$$a_\tau = \frac{v_1^2}{2s_1}.$$

At the middle of the path, where $s_2 = \frac{s_1}{2}$, the velocity v_2 is

$$v_2 = \sqrt{2a_\tau s_2} = \sqrt{a_\tau s_1} = \frac{v_1}{\sqrt{2}}.$$

The normal acceleration at this point of the path is $a_{n2} = \frac{v_2^2}{R} = \frac{v_1^2}{2R}$.

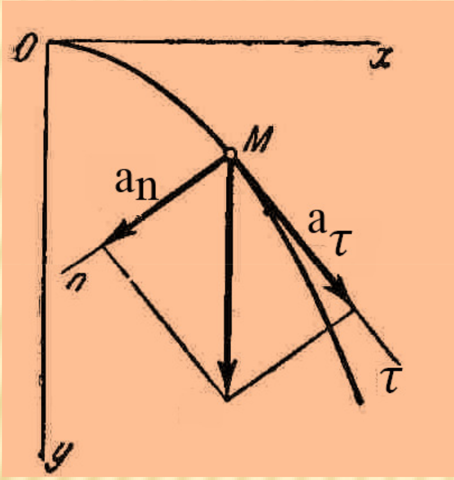
Knowing a_τ and a_{n2} we find: $a_2 = \sqrt{a_\tau^2 + a_{n2}^2} = \frac{v_1^2}{2}\sqrt{\frac{1}{s_1^2} + \frac{1}{R^2}}$.

Substituting the numerical values, we obtain:

$$v_2 \approx 7.1 m/sec, \qquad a_2 = \frac{5}{48} \approx 0.1\, m/sec^2.$$

P r o b l e m. The equations of motion for a particle thrown with a horizontal velocity are $x = v_0 t, \quad y = \frac{1}{2} g t^2$, where v_0 and g are constants.

Determine the path, velocity and acceleration of the particle, its tangential and normal accelerations and the radius of curvature of its path at any point, expressing them in terms of the velocity of the particle at given point.



S o l u t i o n. Determining t from the first equation and substituting its expression into the second, we obtain: $y = \frac{g}{2v_0^2} x^2$. So the path of the particle is a parabola.

Differentiating the equation of motion, we find:

$v_x = \frac{dx}{dt} = v_0; \quad v_y = \frac{dy}{dt} = gt$, whence $v = \sqrt{v_0^2 + g^2 t^2}$.

From the respective equations we have: $a_x = \frac{d^2x}{dt^2} = 0, \quad a_y = \frac{d^2y}{dt^2} = g, \; a = g$.

In the present case the particle has an acceleration of constant magnitude and direction, parallel to y axis. Note that, although $a = const.$, the motion of the particle is not uniformly variable, since the condition for uniformly variable motion is not $a = const.$, but $a_\tau = const$. In this case, we shall find, a_τ is not constant.

Knowing the dependence of v on t, we can find a_τ:

$$a_\tau = \frac{dv}{dt} = \frac{g^2 t}{\sqrt{v_0^2 + g^2 t^2}} = \frac{g^2 t}{v}.$$

But we have $v^2 = v_0^2 + g^2 t^2$, and consequently, $t = \frac{1}{g}\sqrt{v^2 - v_0^2}$.

Substituting this expression of t, we have: $a_\tau = g\sqrt{1 - \frac{v_0^2}{v^2}}$.

It follows that at the initial moment, when $v = v_0$, $a_\tau = 0$, then increasing together with v and, at $v \to \infty, a_\tau \to g$. Thus, in the limit the tangential acceleration approaches the total acceleration g.

To determine a_n, we refer to the equation $a^2 = a_\tau^2 + a_n^2$, whence

$$a_n^2 = a^2 - a_\tau^2 = g^2 - g^2\left(1 - \frac{v_0^2}{v^2}\right) = g^2\frac{v_0^2}{v^2}, \text{ and } a_n = \frac{v_0 g}{v}.$$

Thus, at the initial moment ($v = v_0$), $a_n = g$, decreasing as v increases and in the limit approaching zero.

To determine the radius of curvature of the path, we use the equation $a_n = \frac{v^2}{\rho}$, whence $\rho = \frac{v^2}{a_n} = \frac{v^2}{v_0 g}$.

P r o b l e m. A shaft rotation with a speed of n = 90 rpm decelerates uniformly when the motor is switched off and stops in t_1 = 40 sec. Determine the number of revolutions made by the shaft in this time.

S o l u t i o n. As the rotation is uniformly retarded,

$$\varphi = \omega_0 t + \varepsilon \frac{t^2}{2},\ \omega = \omega_0 + \varepsilon t.$$

The initial angular velocity of the uniformly retarded rotation is that which the shaft had before the motor was switched off. Hence, $\omega_0 = \frac{\pi n}{30}$.

At the instant $t = t_1$, when the shaft stopped, its angular velocity was $\omega_1 = 0$. Substituting these values into equation, we obtain:

$$0 = \frac{\pi n}{30} + \varepsilon t_1 \quad and \quad \varepsilon = -\frac{\pi n}{30 t_1}.$$

If we denote as N the number of revolutions of the shaft in time t_1 (not be confused with n, which is the angular velocity!), the angle of rotation in that time will be $\varphi_1 = 2\pi N$. Substituting the values of ε and φ_1 we obtain

$$2\pi N = \frac{\pi n}{30} t_1 - \frac{\pi n}{60} t_1 = \frac{\pi n}{60} t_1,$$

whence $N = \frac{n t_1}{120} = 30$ revolutions.

P r o b l e m. A flywheel of radius R = 1.2 m rotates uniformly, making n = 90 rpm. Determine the linear velocity and acceleration of a point on the rim of the flywheel.

S o l u t i o n. The linear velocity of such a point is $v = R\omega$, where the angular velocity ω must be expressed in radians per second. In our case

$$\omega = \frac{\pi n}{30} = 3\pi\, sec^{-1}.$$

Hence,

$$v = \frac{\pi n}{30} R \approx 11.3 \frac{m}{sec}.$$

As ω = const., $\varepsilon = 0$, and consequently

$$a = a_n = R\omega^2 = \frac{\pi^2 n^2}{900} R \approx 106.6\, m/sec^2.$$

The acceleration is directed towards the axis of rotation.

P r o b l e m. The equation of the motion of an accelerated flywheel is

$$\varphi = \frac{9}{32}t^3.$$

Determine the linear velocity and acceleration of a point lying at a distance h = 0.8 m from the axis of rotation at the instant when its tangential and normal accelerations are equal.

S o l u t i o n. We determine the angular velocity and angular acceleration of the flywheel:

$$\omega = \frac{d\varphi}{dt} = \frac{27}{32}t^2, \qquad \varepsilon = \frac{d\omega}{dt} = \frac{27}{16}t.$$

The formulas for the tangential and normal acceleration of the point are $a_\tau = h\varepsilon$, and $a_n = h\omega^2$.

Denote the instant when $a_\tau = a_n$ by the symbol t_1. Obviously, at that instant $\varepsilon_1 = \omega_1^2$, or $\frac{27}{16}t_1 = (\frac{27}{32})^2 t_1^4$, whence $t_1^3 = \frac{64}{27}$ and $t_1 = \frac{4}{3}\ sec.$

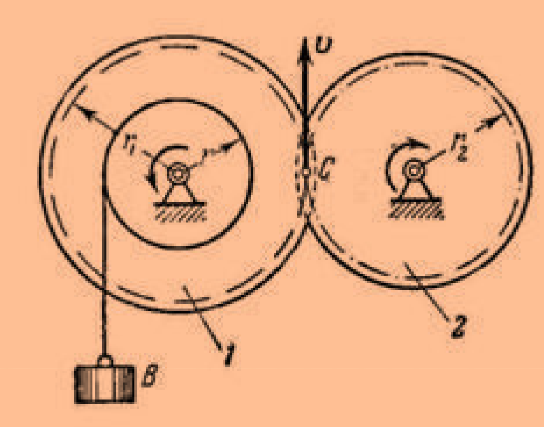
Substituting this value of t_1 in the expressions for ω and ε, we find that at time t_1

$$\omega_1 = \frac{3}{2} sec^{-1}, \qquad \varepsilon_1 = \frac{9}{4}\ sec^{-2}.$$

The required values are thus

$$v_1 = h\omega_1 = 1.2\ m/sec; \quad a_1 = h\sqrt{\varepsilon_1^2 + \omega_1^4} = 1.8\sqrt{2} \approx 2.54\ m/sec^2.$$

Problem. The weight B rotates a shaft of radius r with gear *1* and radius r_1 mounted on it. The weight starts moving from rest with a constant acceleration a. Develop the equation of rotation of the gear 2 of radius r_2 which is meshed with gear *1*.



S o l u t i o n. As the initial velocity of the weight is zero, its velocity at any instant is $v_B = at$. All the points on the surface of the shaft have the same velocity. At the same time, their velocity is $\omega_1 r$, where ω_1 is the angular velocity of both the shaft and gear *1*. Consequently, $\omega_1 r = at, \quad \omega_1 = \frac{at}{r}$.

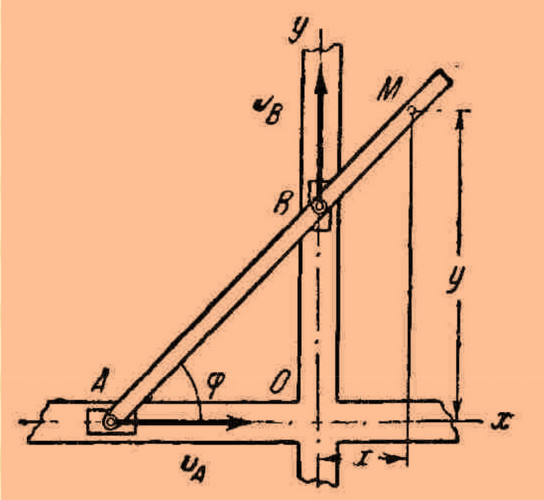
As at point C, where the gears mesh, the linear velocity of both gears must be the same, we have $v_C = \omega_1 r_1 = \omega_2 r_2$, whence $\omega_2 = \frac{r_1}{r_2}\omega_1 = \frac{r_1 a}{r_2 r}t$.

Since $\omega_2 = \frac{d\varphi_2}{dt}$, we have $d\varphi_2 = \frac{r_1 a}{r_2 r}t\,dt$.

Integrating both sides and assume angle $\varphi_2 = 0$ at time $t = 0$, we obtain the equation of uniformly accelerated rotation of gear *2* in the form:

$$\varphi_2 = \frac{r_1 a}{2rr_2}t^2.$$

P r o b l e m. Determine the relation between the velocities of points A and B of the ellipsograph rule if angle φ is given.



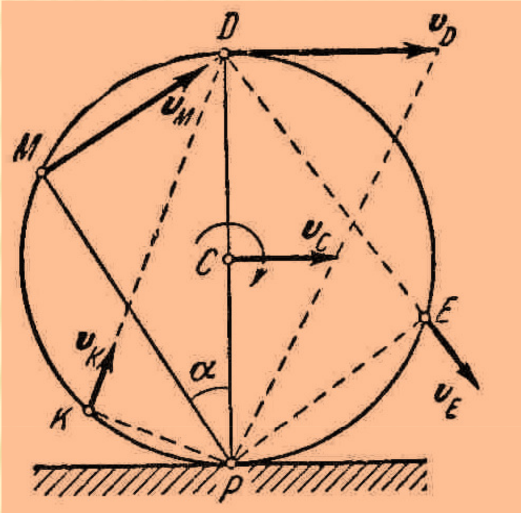
S o l u t i o n. The directions of velocities of point A and B are known. Hence, projecting vectors $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ on AB and applying the theorem of projections of two points, we obtain:

$$v_A \cos\varphi = v_B \cos(90° - \varphi),$$

whence

$$v_A = v_B \tan\varphi\,.$$

P r o b l e m. Determine the velocity of point M on the rim of the rolling wheel by introducing the instantaneous center of zero velocity.



S o l u t i o n. The point of contact P of the wheel is the instantaneous center of zero velocity, as $v_P = 0$. Consequently, $\boldsymbol{v_M} \perp PM$. As the right angle PMD rests on the diameter, the velocity vector $\boldsymbol{v_M}$ of any point of the rim passes through point D.

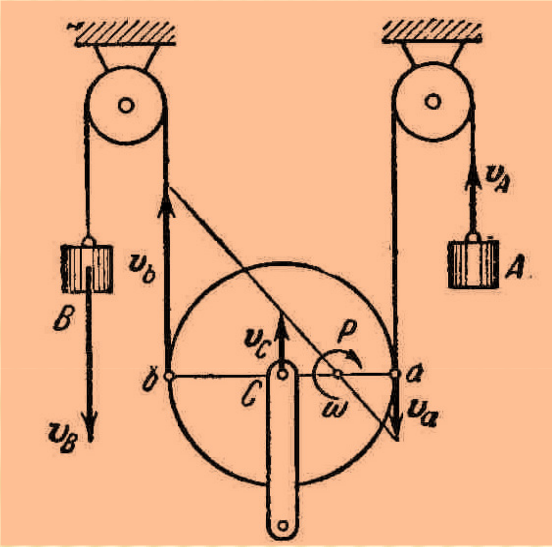
Writing the proportion $\frac{v_M}{PM} = \frac{v_C}{PC}$ and noting that $PC = R$ and $PM = 2R\cos\alpha$, we find

$$v_M = 2v_C \cos\alpha \,.$$

The further point M is from P, the grater its velocity. The upper end D of the vertical diameter has the maximum velocity $v_P = 2v_C$. The angular velocity of the wheel is $\omega = \frac{v_C}{PC} = \frac{v_C}{R}$.

The velocities are similarly distributed for all cases of a wheel or gear rolling along a cylindrical surface.

P r o b l e m. Determine the velocity of the centre C of the pulley of radius r and its angular velocity if load A is moving up with a velocity v_A and load B is moving down with a velocity v_B. The thread does not slip and all its sections are vertical.



S o l u t i o n. As the thread does not slip on the pulley, the velocities of points a and b of the pulley are equal in magnitude to the velocities of the loads, i.e., $v_a = v_A$ and $v_b = v_B$.

Knowing the velocities of points a and b and assuming for convenience that $v_B > v_A$, we can determine the position of the instantaneous centre of zero velocity P of the pulley. The velocity of the centre of pulley C is denoted by the vector $\boldsymbol{v_C}$.

We develop the equations: $\omega = \frac{|\boldsymbol{v_b} + (-\boldsymbol{v_a})|}{ab}$ $\quad \omega = \frac{|\boldsymbol{v_b} - \boldsymbol{v_c}|}{bC}$,

whence, as $ab = 2r$ and $bC = r$, we obtain $\omega = \frac{v_B + v_A}{2r}$, $\quad v_C = \frac{v_B - v_A}{2}$.

At $v_B > v_A$ the centre moves up; if $v_B < v_A$ it moves down; at $v_B = v_A$, $v_C = 0$.

The values of ω and v_C for the case of both loads lowering can be found by substituting $-v_A$ for v_A in the equations.

P r o b l e m. Link OA rotates about axis O with an angular velocity ω_{OA}; with it moves gear *1* which rolls around the fixed gear *2*. The radii of the two gears are both equal to r. Hinged to gear *1* is a connecting rod BD of length l, attached to which is a rockshaft DC. Determine the angular velocity ω_{BD} of the connecting rod for the instant when it is perpendicular to link OA, if at that instant angle $BDC = 45°$.

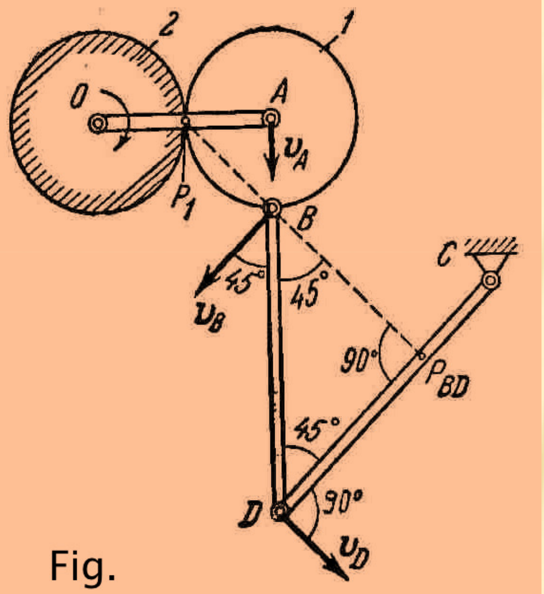


Fig.

S o l u t i o n. Let us determine the velocity of point B from the fact that it also belongs to gear *1*, for which the velocity is known to be $v_A = \omega_{OA} 2r$, $(\boldsymbol{v_A} \perp OA)$ and the instantaneous center of zero velocity is at P_1. Consequently, $v_B \perp PB$, and from the theorem of the projections of velocities $v_B \cos 45° = v_A$, whence $v_B = v_A\sqrt{2} = 2r\omega_{OA}\sqrt{2}$.

Now we know the velocity $\boldsymbol{v_B}$ of a point of the connecting rod and the direction of velocity $\boldsymbol{v_D}$, $(\boldsymbol{v_D} \perp DC)$.

Erecting perpendiculars to $\boldsymbol{v_B}$ and $\boldsymbol{v_D}$, we obtain the instantaneous centre P_{BD} of the connecting rod: $BP_{BD} = l\frac{\sqrt{2}}{2}$, whence $\omega_{BD} = \frac{v_B}{BP_{BD}} = 4\frac{r}{l}\omega_{OA}$.

P r o b l e m. A gear *1* and crank *OA* are mounted independently of each other on an axle *O*. The crank rotates with an angular velocity ω_{OA}. Fixed to the connecting rod *AB* with its center at *A* is gear *2*. The crank *OA* carries axis *A* of the gear *2*, the connecting rod passes through the rocker slide *C*. The radii of gears *1* and *2* are both equal to *r*. Determine the angular velocity ω_1 of gear *1* at the instant when $OA \perp OC$, if $\angle ACO = 30°$.

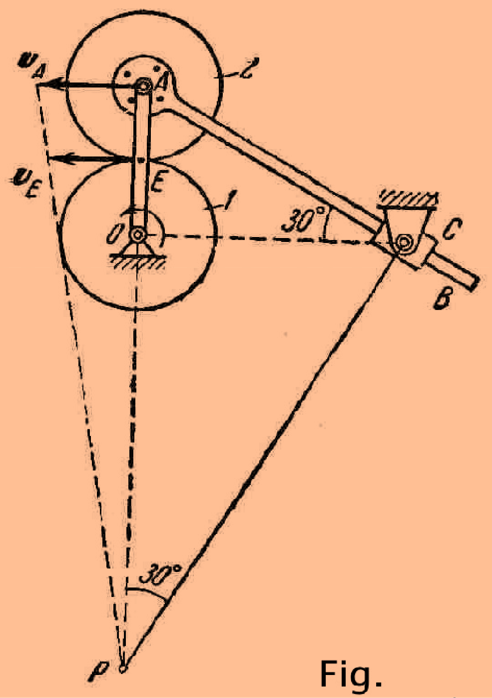


Fig.

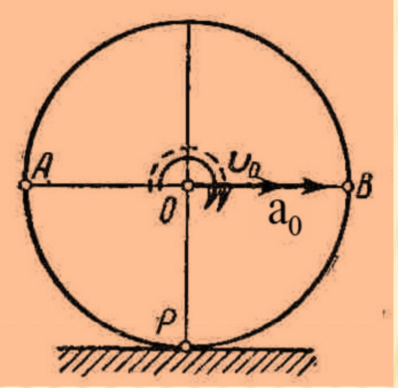
S o l u t i o n. For gear *2* we know the direction and magnitude of the velocity of point *A*: $\boldsymbol{v_A} \perp OA, \quad v_A = \omega_{OA} 2r.$

Besides, we know the direction of velocity $\boldsymbol{v}_E$.

Gear *2* and the connecting rod are actually one body for which we know the direction of the velocity of point *C*: vector $\boldsymbol{v}_C$ is directed along *CA*, as at point *C* the rod can only slide in the rocker. By erecting the perpendiculars to $\boldsymbol{v}_A$ and $\boldsymbol{v}_C$, we obtain the instantaneous center of zero velocity *P* of the body *BAE*.

From the statement of the problem $\angle ACO = 30°$, hence $\angle CPA = 30°$. Therefore *AC* = *2AO* = 4*r*, *PA* = *2AC* = 8*r*, *PE* = 7*r*, and from the proportion $\frac{v_E}{PE} = \frac{v_A}{PA}$ we find that $v_E = \frac{7}{8} v_A = \frac{7}{4} r\omega_{OA}$, whence $\omega_1 = \frac{v_E}{OE} = \frac{7}{4}\omega_{OA}$.

P r o b l e m. The center O of a wheel of radius R = 0.2 m rolling along a straight rail has at a given instant a velocity v_0 = 1 m/sec and an acceleration $a_0 = 2$ m/sec^2. Determine the acceleration of the point B lying at the end of diameter AB perpendicular to OP and the acceleration of the point P coincident with the instantaneous center of zero velocity.



S o l u t i o n. The point of contact P is the instantaneous center of zero velocity; hence the angular velocity of the wheel is

$$\omega = \frac{v_0}{PO} = \frac{v_0}{R}.$$

As the quantity $PO = R$ is constant for any position of the wheel, by differentiating the equation with respect to time we obtain

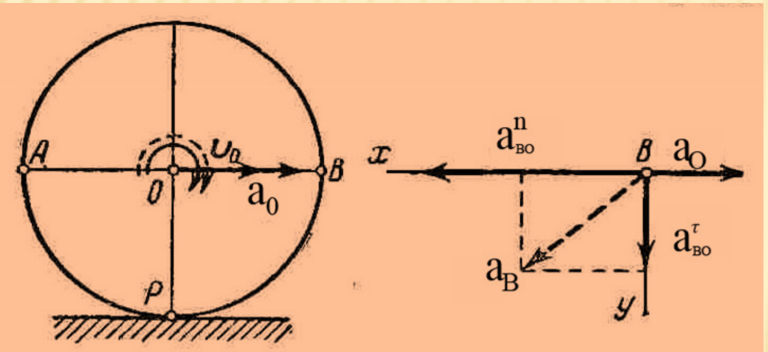
$$\frac{d\omega}{dt} = \frac{1}{R}\frac{dv_0}{dt} \quad or \quad \varepsilon = \frac{a_0}{R}.$$

The signs of ε and ω are the same, therefore the rotation of the wheel is accelerated.

It should not be assumed that v_0 is constant only because the given value of v_0 = 1 m/sec. This value is for the given instant, and it changes with time, since $a_0 \neq 0$. In this case $\frac{dv_0}{dt} = a_0$, as the motion of point O is rectilinear. In the general case $\frac{dv_0}{dt} = a_{0\tau}$.

As point O is our pole, we have $\boldsymbol{a_B} = \boldsymbol{a_0} + \boldsymbol{a_{BO}^{\tau}} + \boldsymbol{a_{BO}^{n}}$.

In our case $BO = R$, and $a_{BO}^{\tau} = BO \cdot \varepsilon = a_o = 2\, m/sec^2$, $a_{BO}^{n} = BO \cdot \omega^2 = \frac{v_0^2}{R} = 5\, m/sec^2$.



Drawing axes Bx and By, we find that

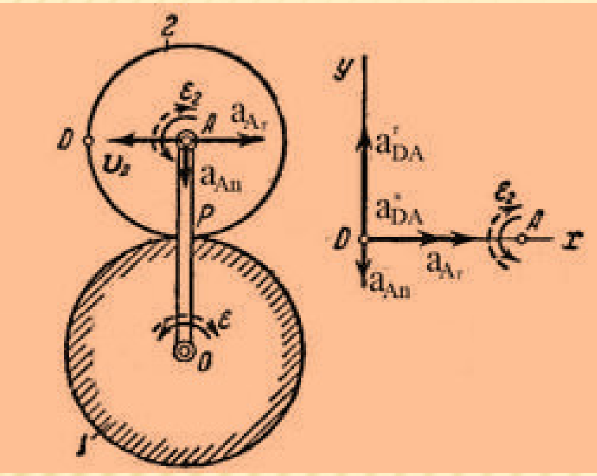
$$a_{Bx} = a_{BO}^{n} - a_O = \frac{3m}{sec^2},$$

$$a_{By} = a_{BO}^{\tau} = \frac{2m}{sec^2}.$$

Whence $a_B = \sqrt{a_{Bx}^2 + a_{By}^2} = \sqrt{13} \approx 3.6\, m/sec^2$.

Similarly we can easily find that the acceleration of point P is $a_P = a_{BO}^{\tau} = 5\, m/sec^2$ and is directed along PO. Thus, the acceleration of point P, whose velocity at the given instant is zero, is not zero.

P r o b l e m. Gear *1* of radius $r_1 = 0.3$ m is fixed; rolling around it is gear *2* of radius $r_2 = 0.2$ m mounted on link *OA*. The link turns about axis *O* and has at the given instant an angular velocity $\omega = 1\ sec^{-1}$ and an angular acceleration $\varepsilon = -4\ sec^{-2}$. Determine the acceleration of point the *D* on the rim of the moving gear at the given instant (radius *AD* is perpendicular to the link).



S o l u t i o n. From the statement of the problem it is easy to determine the velocity $\boldsymbol{v_A}$ and acceleration $\boldsymbol{a_A}$ of the point *A* of the gear, which we take as the pole.

Knowing ω and ε of the link, we obtain

$$v_A = OA \cdot \omega = 0.5\ m/sec^2,$$
$$a_{A\tau} = OA \cdot \varepsilon = -2\ m/sec^2,$$
$$a_{An} = OA \cdot \omega^2 = 0.5\ m/sec^2.$$

The point of contact *P* is the instantaneous center of gear *2;* consequently, the angular velocity of gear *2* is $\omega_2 = \frac{v_A}{AP} = \frac{v_A}{r_2}$; $\omega_2 = 2.5\ sec^{-1}$.

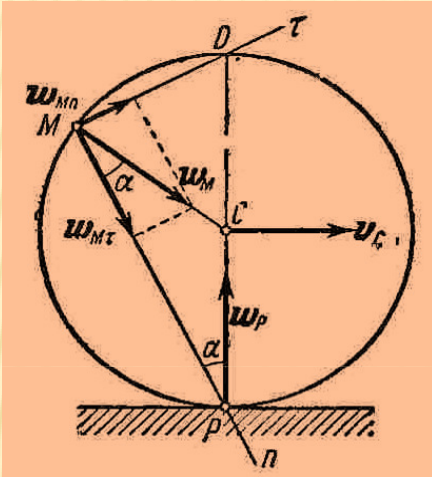
The acceleration of point *D* is $\boldsymbol{a_D} = \boldsymbol{a_{A\tau}} + \boldsymbol{a_{An}} + \boldsymbol{a_{DA}^{\tau}} + \boldsymbol{a_{DA}^{n}}$.

In our case $DA = r_2$, $a_{DA}^{\tau} = DA \cdot \varepsilon_2 = -2\ m/sec^2$, $a_{DA}^{n} = DA \cdot \omega_2^2 = 1.25\ m/sec^2$.

Drawing axes *Dx* and *Dy*, we find that $a_{Dx} = |a_{A\tau}| + a_{DA}^{n} = 3.25\ m/sec^2, a_{Dy} =$

$|a_{DA}^{\tau}| - a_{An} = 1.5\ m/sec^2$, whence $\boldsymbol{a_D} = \sqrt{a_{Dx}^2 + a_{Dy}^2} \approx 3.58\ m/sec^2$.

P r o b l e m. A wheel rolls along a straight rail so that the velocity $\boldsymbol{v}_C$ of its center C is constant. Determine the acceleration of a point M on the rim of the wheel.



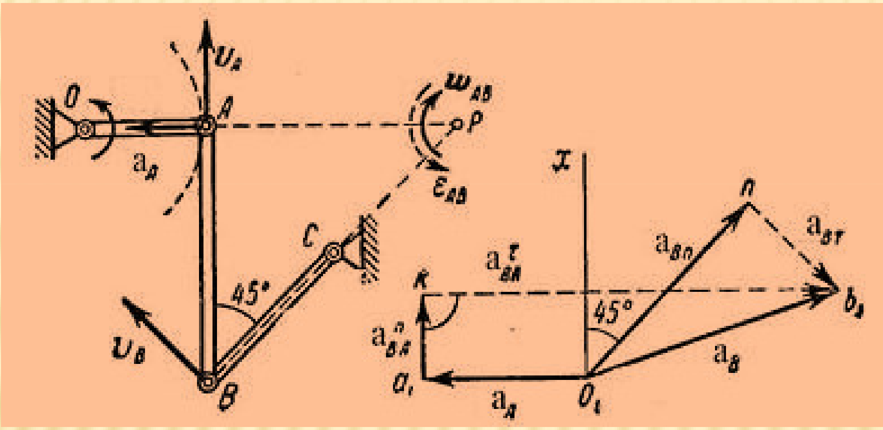
S o l u t i o n. As $v_C = const.$, then point C is the instantaneous center of zero acceleration. The instantaneous center of zero velocity is at P.

Consequently, $\omega = \frac{v_C}{PC} = \frac{v_C}{R} = const.$,

$$\varepsilon = \frac{d\omega}{dt} = 0, \qquad \tan\mu = \frac{\varepsilon}{a^2} = 0, \qquad \mu = 0$$

Thus, the acceleration of any point M on the rim (including P) is equal to v_C^2/R and is directed towards the center of the wheel, since angle $\mu = 0$.

P r o b l e m. Attached to a crank OA rotating uniformly about axis O with an angular velocity $\omega_{OA} = 4 sec^{-1}$ is a connecting rod AB hinged to a rockshaft BC. The given dimensions are: $OA = r = 0.5$ m, AB = $2r$, $BC = r\sqrt{2}$. In the position shown in the diagram, $\angle OAB = 90°$ and $\angle ABC = 45°$. Determine for this position the acceleration of point B of the connecting rod and the angular velocity and angular acceleration of the rockshaft BC.



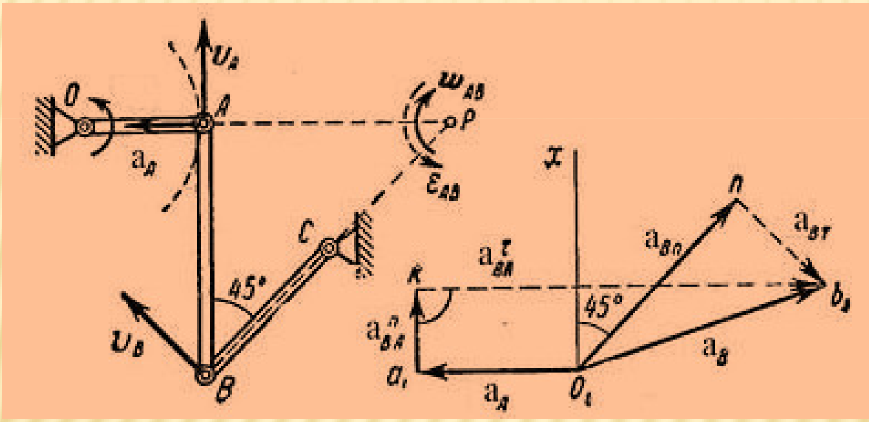
S o l u t i o n. Considering the motion of the connecting rod AB, we take point A as the pole. As $\omega_{OA} = const.$, we obtain: $v_A = r\omega_{OA} = 2\, m/sec$, $a_A = a_{An} = r\omega_{OA}^2 = 3\, m/sec^2$.

We know the path of point B of the connecting rod (a circle of radius BC). Hence, knowing the direction of $\boldsymbol{v_B}$ ($\boldsymbol{v_B} \perp BC$), we can locate the instantaneous center of zero velocity P of the rod. It is evident that $AP = AB = 2r$. Therefore, $\omega_{AB} = \frac{v_A}{AP}$ or $\omega_{AB} = \frac{\omega_{OA}}{2} = 2\, sec^{-1}$.

Knowing ω_{AB}, we have $a_{BA}^{n} = AB \cdot \omega_{AB}^{2} = 4\, m/sec^{2}$.

Knowing the path of point B, we can determine its normal acceleration $\boldsymbol{a_{Bn}}$. For this, applying the theorem of projections (or the instantaneous center of zero velocity P), we first determine the velocity v_B. We have $v_B \cos 45° = v_A$, whence $v_B = v_A\sqrt{2}$. Therefore, $a_{Bn} = \frac{v_B^2}{BC} = \frac{2v_A^2}{r\sqrt{2}} = 8\sqrt{2}\, m/sec^2$.

Then $\boldsymbol{a_A} + \boldsymbol{a_{BA}^{n}} + \boldsymbol{a_{BA}^{\tau}} = \boldsymbol{a_{Bn}} + \boldsymbol{a_{B\tau}}$.



Now draw coordinate axis O_1x perpendicular to the unknown vector a_{BA}^{τ} and project on it both sides of vector equation (e). We obtain $a_{BA}^{n} = a_{Bn}\cos 45° - |a_{B\tau}|\cos 45°$, whence

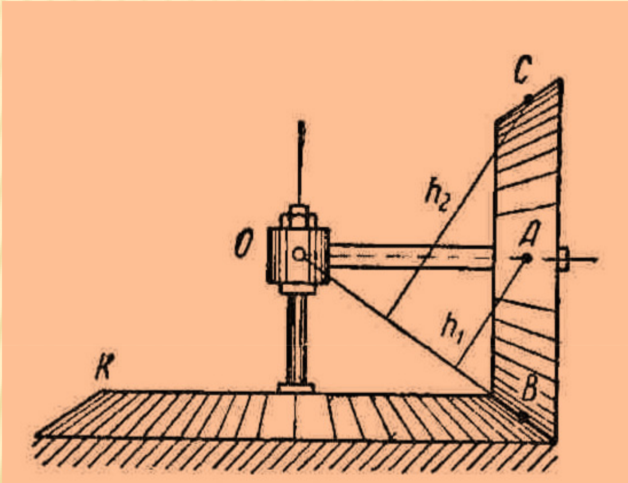
$$|a_{B\tau}| = a_{Bn} - a_{BA}^{n}\sqrt{2} = $$
$$= 8\sqrt{2} - 4\sqrt{2} = 4\sqrt{2}.$$

And finally $a_B = \sqrt{a_{B\tau}^2 + a_{Bn}^2} = 4\sqrt{10} = 12.65\, \frac{m}{sec^2}$.

Then $\omega_{BC} = \frac{v_B}{BC}$, $\quad |\varepsilon_{BC}| = \frac{|a_{B\tau}|}{BC}$.

Solving them, we obtain: $\omega_{BC} = 4\, sec^{-1}, \varepsilon_{BC} = -8\, sec^{-2}$ (the minus shows that $\boldsymbol{a_{B\tau}}$ is directed opposite to $\boldsymbol{v_B}$).

P r o b l e m. Determine the velocities of point B and C of the bevel wheel if the velocity $\boldsymbol{v_A}$ of the wheel center A along its path is known. The wheel runs without slipping on the fixed conic surface K.



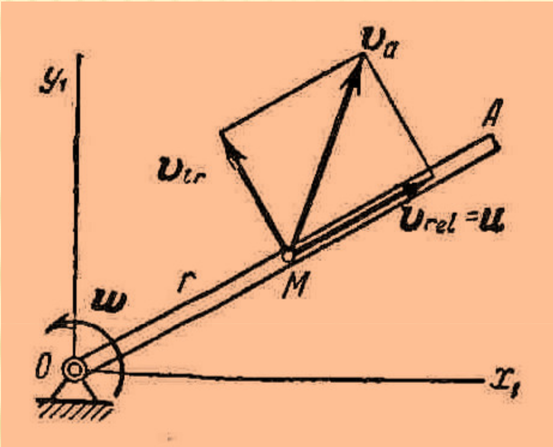
S o l u t i o n. The wheel rotates about a fixed point O. As it runs without slipping, the points of the wheel on line OB must have the same velocity as the points of surface K, i.e., zero, and OB is the instantaneous axis of rotation of the wheel.

Therefore $v_A = v_A/h_1$, where ω is the angular velocity of the wheel in its motion about axis OB, and h_1 is the distance of A from that axis.

Hence, $\omega = v_A/h_1$.

The velocity v_C of point C is ωh_2 where h_2 is the distance of C from OB. As in this case $h_2 = 2h_1$, $v_C = 2v_A$. From the point B, which is on the instantaneous axis of rotation, $v_B = 0$.

P r o b l e m. Point M moves in a straight line along OA with a velocity u, while OA itself turns in the plane Ox_1y_1 round O with an angular velocity ω. Find the velocity of point M relative to the axes Ox_1y_1 expressed as a function of the distance $OM = r$.



S o l u t i o n. Consider the motion of point M as a resultant motion consisting of its relative motion along OA and its motion together with OA.

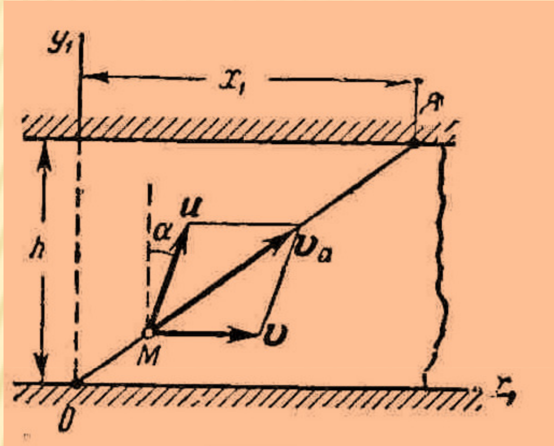
The velocity u along OA is the relative velocity of the point.

The rotational motion of OA about O is, for the point M, the motion of transport, and the velocity of the point of OA with which M coincides at the given instant is the transport velocity $\boldsymbol{v}_{tr}$.

As this point of OA moves along a circle of radius $OM = r$, $v_{tr} = r\omega$ and is perpendicular to OM.

Constructing a parallelogram with vectors $\boldsymbol{u}$ and $\boldsymbol{v}_{tr}$ as its sides, we obtain the absolute velocity $\boldsymbol{v}_a$ of M relative to the axes Ox_1y_1. As $\boldsymbol{u}$ and $\boldsymbol{v}$ are mutually perpendicular, in magnitude $v_a = \sqrt{u^2 + \omega^2 r^2}$.

P r o b l e m**.** The current of a river of width h has a constant velocity v. A man can row a boat in motionless water with a velocity u. Determine the direction he should take in order to cross the river in the least possible time and the point where he will reach the opposite bank.



S o l u t i o n. Assume that the boat has started from point O. Assume further that the rower steers his boat at a constant angle α to axis Oy_1. Then the absolute velocity $\boldsymbol{v}_A$ of the boat is compounded of the relative velocity $\boldsymbol{v}_{rel}$ imparted to it by the rower ($\boldsymbol{v}_{rel} = \boldsymbol{u}$) and the transport velocity $\boldsymbol{v}_{tr}$, which is the velocity of the stream($\boldsymbol{v}_{tr} = \boldsymbol{v}$):$\boldsymbol{v}_a = \boldsymbol{v}_{rel} + \boldsymbol{v}_{tr} = \boldsymbol{u} + \boldsymbol{v}$.

The projections of the absolute velocity on the coordinate axes are

$$v_{ax_1} = u \sin\alpha + v; \quad v_{ay_1} = u\cos\alpha\,.$$

As both projections are constant, the displacements of the boat along the coordinate axes are $x_1 = (u\sin\alpha + v)t;\quad y_1 = (u\cos\alpha)t.$

When the boat reaches the opposite bank, $y_1 = h$, whence the duration of the crossing is $t_1 = \dfrac{h}{u\cos\alpha}$.

Obviously t_1 will have the least value when $\cos\alpha = 1$, i.e., when $\alpha = 0$.

Consequently, in order to cross the river in the shortest time, the rower should steer his boat perpendicular to the bank. This time is:

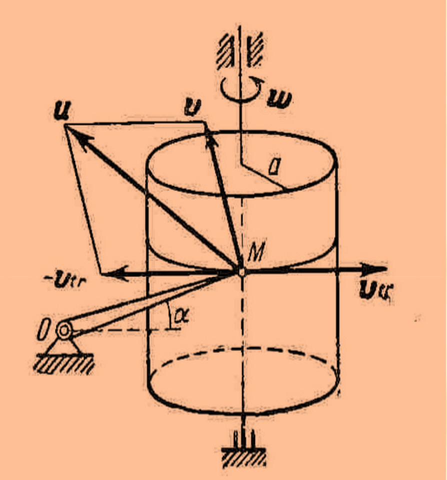
$$t_{min} = \frac{h}{u}.$$

Assuming $\alpha = 0$ and $t = t_{min}$ in the expression for x_1, we have

$$x_1 = \frac{v}{u}h.$$

Thus, the boat will reach the other bank at a point B at a distance x_1 downstream from Oy_1 directly proportional to v and h and inversely proportional to u.

P r o b l e m. At a given instant, the arm *OM* of a recording mechanism makes an angle α with the horizontal and the pencil *M* has a velocity $\boldsymbol{v}$ directed perpendicular to *OM*. The drum with the paper rotates about a vertical axis with an angular velocity ω. Determine the velocity $\boldsymbol{u}$ of the pencil on the paper if the radius of the drum is a.

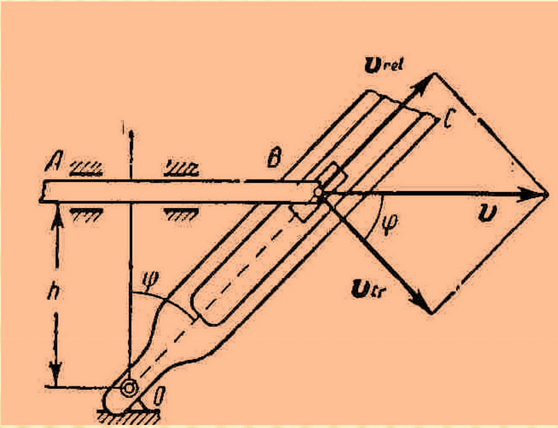


S o l u t i o n. The absolute velocity of the pencil is $\boldsymbol{v_a} = \boldsymbol{v}$. Velocity $\boldsymbol{v}$ can be regarded as the geometrical sum of the velocity of the pencil relative to the paper (i.e., the required velocity $\boldsymbol{u}$) and the transport velocity $\boldsymbol{v_{tr}}$, which is equal to the velocity of the point of the paper with which the pencil coincides at the given moment; its magnitude is $v_{tr} = \omega a$.

From the theorem of the composition of velocities we have $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{v_{tr}}$, whence $\boldsymbol{u} = \boldsymbol{v} + (-\boldsymbol{v_{tr}})$.

Constructing a parallelogram with vectors $\boldsymbol{v}$ and $(-\boldsymbol{v_{tr}})$ as its sides, we obtain the required velocity $\boldsymbol{u}$. As the angle between $\boldsymbol{v}$ and $(-\boldsymbol{v_{tr}})$ is $90° - \alpha$, in magnitude $u = \sqrt{v^2 + \omega^2 a^2 + 2v\omega a \sin\alpha}$.

P r o b l e m. End B of a horizontal rod AB is hinged to a block sliding along the slots of a rocker OC and turns the latter round axis O. The distance from O to AB is h. Find the dependence of the angular velocity of the rocker on the velocity v of the rod and angle φ.



S o l u t i o n. The absolute velocity of the slide block equals the velocity $\boldsymbol{v}$ of the rod. It can be regarded as compounded of the relative velocity $\boldsymbol{v}_{rel}$ of the block in its motion in the slots of the rocker and the transport velocity $\boldsymbol{v}_{tr}$, which is the velocity of the point of the rocker with which the block coincides at the given time.

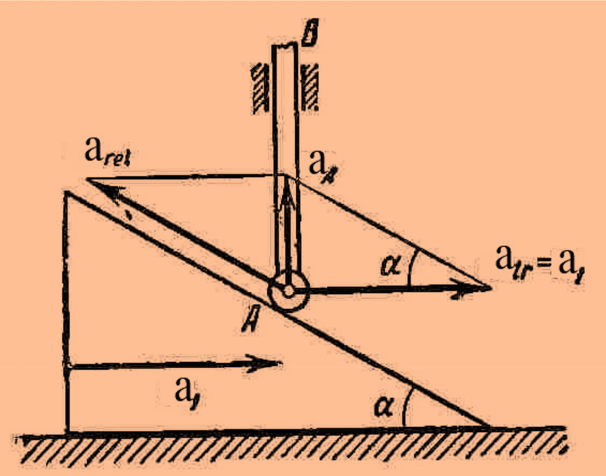
The direction of these velocities are along OB and perpendicular to OB, respectively.

We obtain $\boldsymbol{v}_{rel}$ and $\boldsymbol{v}_{tr}$ by resolving velocity $\boldsymbol{v}$ along them. From the parallelogram we find that in magnitude $v_{tr} = v\cos\varphi$.

But, on other hand, the transport velocity $v_{tr} = \omega \cdot OB = \omega\frac{h}{\cos\varphi}$, where ω is the angular velocity of the rocker.

Equating expressions of v_{tr}, we obtain the angular velocity: $\omega = \frac{v}{h}cos^2\varphi$.

P r o b l e m. A wedge moving horizontally with an acceleration $\boldsymbol{a_1}$ pushes up a rod moving in vertical slides. Determine the acceleration of the rod if the angle of the wedge is α.

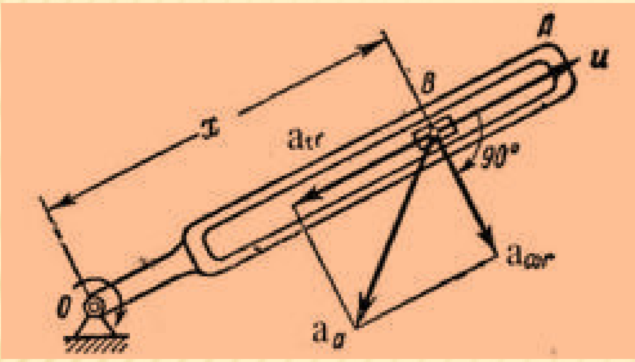


S o l u t i o n. The absolute acceleration $\boldsymbol{a_A}$ of point A is directed vertically up. It can be regarded as consisting of a relative acceleration $\boldsymbol{a_{rel}}$ directed along the side of the wedge and a transport acceleration $\boldsymbol{a_{tr}}$, which is equal to the acceleration of the wedge $\boldsymbol{a_1}$.

As the motion of transport of the wedge is translation, by drawing a parallelogram and taking into account that $\boldsymbol{a_{tr}} = \boldsymbol{a_1}$, we obtain $\boldsymbol{a_A} = \omega_1 \tan \alpha$.

Which is the acceleration of the rod.

P r o b l e m. The rocker OA turns with a constant angular velocity ω about axis O. A block B slides along the slots with a constant relative velocity u. Determine the dependence of the absolute acceleration of the block on its distance x from O.



S o l u t i o n. Stopping the rocker, we find that the relative motion of the block along it is uniform and rectilinear; consequently $a_{rel} = 0$.

For the block, the motion of the rocker is that of transport.

Consequently, the transport acceleration $\boldsymbol{a}_{tr}$ of the block is equal to the acceleration of the point of the rocker, with which the block coincides.

Since that point of the rocker is moving in a circle of radius $OB = x$ and $\omega = const.$, vector $\boldsymbol{a}_{tr} = \boldsymbol{a}_{tr}^{n}$ and is directed along BO. In magnitude $a_{tr} = a_{tr}^{n} = \omega^2 x$.

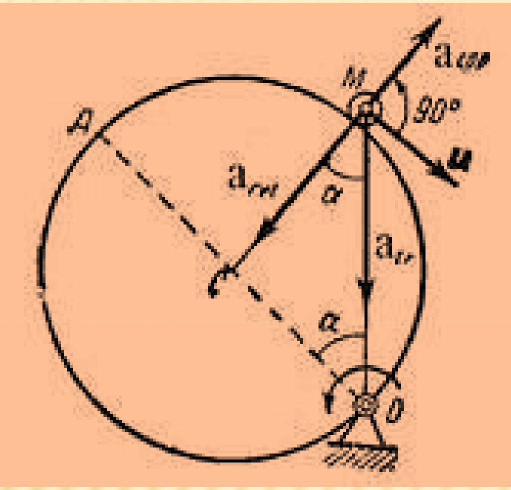
The Coriolis acceleration $a_{cor} = 2\omega u$. By turning the vector of the relative velocity $\boldsymbol{u}$ about point B through a right angle in the direction of the rotation of transport (clockwise), we obtain the direction of $\boldsymbol{a}_{cor}$.

From Coriolis theorem, $\boldsymbol{a}_a = \boldsymbol{a}_{rel} + \boldsymbol{a}_{tr} + \boldsymbol{a}_{cor}$.

In the present case $\boldsymbol{a}_{rel} = 0$ and $\boldsymbol{a}_{cor}$ is perpendicular to $\boldsymbol{a}_{tr}$.

Consequently, $a_a = \sqrt{a_{tr}^2 + a_{cor}^2} = \omega\sqrt{\omega^4 x^2 + 4u^2}$.

P r o b l e m. The eccentric is a circular disc of radius R rotating with a uniform angular velocity ω about axis O through the rim of the disc. Sliding from point A along the disc with a constant relative velocity u is a pin M. Determine the absolute acceleration of the pin at any time t.



S o l u t i o n. At time t the pin is at a distance $s = \overset{\smile}{AM} = ut$ from A. Consequently, at the instant angle $AOM = \alpha$ will be

$$\alpha = \frac{s}{2R} = \frac{u}{2R}t,$$

as α is equal to half the central angle ACM. Stopping the motion of the disc at time t, we find that the relative motion of the pin is along a circle of radius R.

As $v_{rel} = u = const.$, $a_{rel}^{\tau} = \frac{du}{dt} = 0$; $\quad a_{rel}^{n} = \frac{u^2}{R}$.

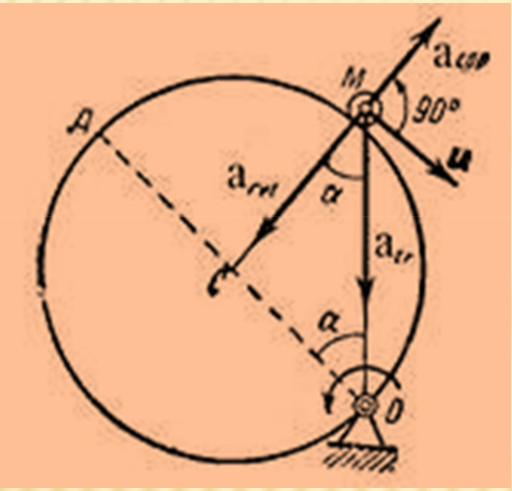
The vector $\boldsymbol{a}_{rel} = \boldsymbol{a}_{rel}^{n}$ is directed along the radius MC.

For the pin the motion of the disc is that of transport. Hence, the transport acceleration $\boldsymbol{a}_{tr}$ of the pin is equal to the acceleration of the point of the disc with which it coincides at the given time. This point moves in a circle of radius $OM = 2R\cos\alpha$. For the disc, $\omega = const.$, hence $\varepsilon = 0$, and $a_{tr}^{\tau} = OM \cdot \varepsilon = 0$; $\quad a_{tr}^{n} = OM \cdot \omega^2 = 2R\omega^2 \cos\alpha$.

Vector $\boldsymbol{a}_{tr} = \boldsymbol{a}_{tr}^{n}$ is directed along *MO*.

As the motion is in one plane, $a_{cor} = 2\omega u$.

The direction of $\boldsymbol{a}_{cor}$ is found by turning vector $\boldsymbol{v}_{rel} = \boldsymbol{u}$ round point *M* through 90° in the direction of the motion of transport (counter-clockwise).



The absolute acceleration of the pin is

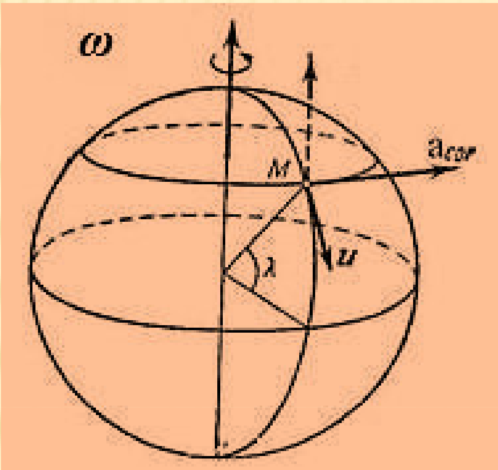
$$\boldsymbol{a}_a = \boldsymbol{a}_{rel} + \boldsymbol{a}_{tr} + \boldsymbol{a}_{cor}.$$

In this case vectors $\boldsymbol{a}_{rel}$ and $\boldsymbol{a}_{cor}$ are collinear and can be replaced by a collinear vector $\boldsymbol{a}_1$ of magnitude $\boldsymbol{a}_1 = \boldsymbol{a}_{rel} - \boldsymbol{a}_{cor}$.

Adding vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_{tr}$ according to the parallelogram law, we obtain finally

$$a_a = \sqrt{a_{tr}^2 + (a_{rel} - a_{cor})^2 + 2a_{tr}(a_{rel} - a_{cor})\cos\alpha}\,.$$

P r o b l e m. A body in the Northern Hemisphere is translated from North to South along a meridian with a velocity $v_{rel} = u\, m/sec$. Determine the magnitude and direction of the Coriolis acceleration of the body at latitude λ.



S o l u t i o n. Neglecting the dimensions of the body, we treat it as a particle. The relative velocity of the body u makes an angle α with the earth's axis. Consequently, $a_{cor} = 2\omega u \sin\lambda$, where ω is the angular velocity of the earth's rotation.

Thus, the Coriolis acceleration is greatest at the Pole, where $\lambda = 90°$.

As the body approaches the equator, the value of a_{cor} decreases, till it reaches zero at the equator, where the vector $\boldsymbol{v}_{rel} = \boldsymbol{u}$ is parallel to the axis of rotation of the earth.

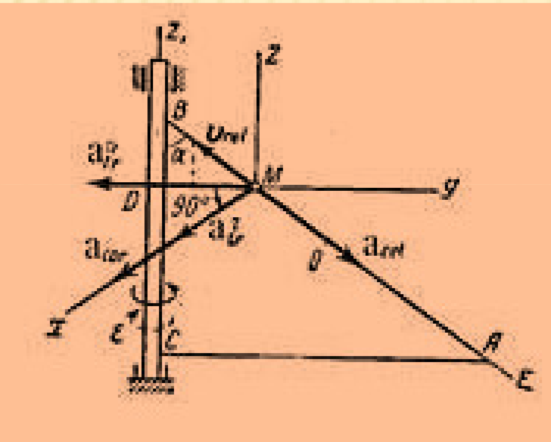
The direction of $\boldsymbol{a}_{cor}$ is found by the rule of a vector product.
As $\boldsymbol{a}_{cor} = \boldsymbol{2}(\boldsymbol{\omega} \times \boldsymbol{u})$ we find that vector $\boldsymbol{a}_{cor}$ is perpendicular to the plane through vectors u and ω, i.e., perpendicular to the meridian plane, and is directed eastwards, from where the shortest turn from vector $\boldsymbol{\omega}$ to vector $\boldsymbol{u}$ is seen anticlockwise.

The question of how the Coriolis acceleration affects the motion of bodies at the earth's surface is studied in the course of dynamics. However, from the formula obtained it can be seen that the value of a_{cor} is usually small, as the angular velocity of rotation of the earth is small:

$$\omega \approx \frac{2\pi}{24 \cdot 3{,}600} sec^{-1}.$$

It is apparent, therefore, that for motions with small velocities the Coriolis acceleration can, for all practical purposes, be neglected.

P r o b l e m. The hypothenuse of the right-angled triangle *ABC* is *AB* = 2a = 20 cm, and $\angle CBA = \alpha = 60°$. The triangle rotates about axis Cz_1 according to the law $\varphi = 10t - 2t^2$. Particle *M* oscillates along *AB* about its middle *O*, its equation of motion being $\xi = a\cos(\frac{\pi}{3}t)$ (axis $O\xi$ is directed along *OA*). Determine the absolute acceleration of the particle *M* at time t_1 = 2 sec.



S o l u t i o n. 1) Determine the position of *M* on its relative path *AB* at time t_1. From the equation of the motion we have

$$\xi_1 = a\cos\left(\frac{2\pi}{3}\right) = -\frac{a}{2},$$

i.e., at time t_1 the particle *M* is at the middle of segment *OB*.

As the relative motion is rectilinear, $v_{rel} = \frac{d\xi}{dt} = -\frac{\pi}{3}a\sin(\frac{\pi}{3}t)$.

At time t_1 = 1 sec, $v_{rel} = -\frac{\pi}{6}a\sqrt{3}$; $|v_{rel_1}| = \frac{5}{3}\pi\sqrt{3}\ cm/sec.$

Differentiating, we obtain $\omega = \frac{d\varphi}{dt} = 10 - 4t, \omega_1 = 2\ sec^{-1}, \varepsilon = \frac{d\omega}{dt} = -4\ sec^{-2}$

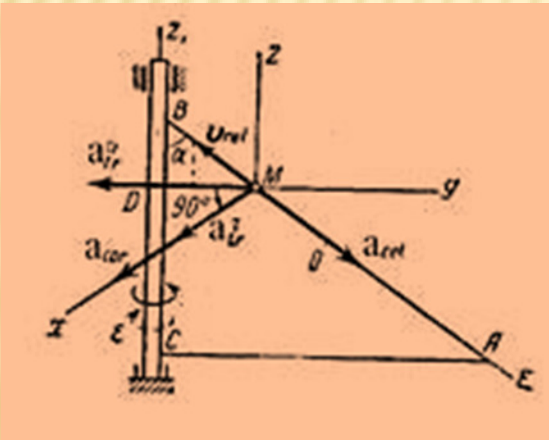
As the relative motion is rectilinear $a_{rel} = \frac{dv_{rel}}{dt} = -\frac{\pi^2}{9} a \cos\left(\frac{\pi}{3} t\right)$.

At time t_1 = 2 sec, $a_{rel} = \frac{\pi^2}{18} a = \frac{5}{9}\pi^2\ cm/sec^2$

For the particle *M* the motion of the triangle is that of transport, and the transport acceleration of *M* is equal to the acceleration of the point of the triangle with which *M* coincides at the given time. This point of the triangle moves in a circle of radius *MD* = *h*, and at time t_1 = 2 sec, $h_1 \frac{a}{2} \sin\alpha = 5\frac{\sqrt{3}}{2}\ cm$.

Thus, $a_{tr}^{\tau} = \varepsilon h = -10\sqrt{3}\ cm/sec^2$; $a_{tr}^{n} = \omega^2 h = 10\sqrt{3}\ cm/sec^2$.

Vector $\boldsymbol{a}_{tr}^{\tau}$ is normal to plane *ABC* in the direction opposite to that of the rotation of the triangle. Vector $\boldsymbol{a}_{tr}^{n}$ is directed along *MD* towards the axis of rotation Cz_1.

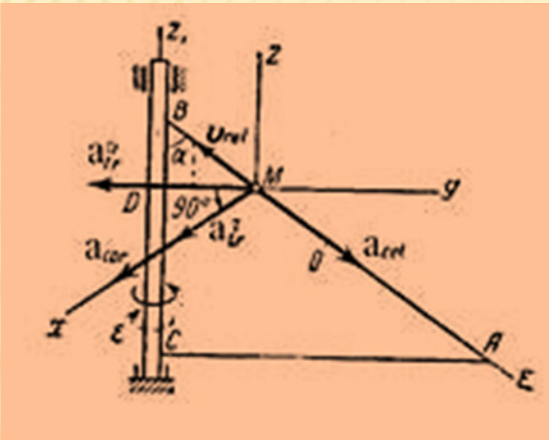


At time t_1 = 2 sec, $a_{cor} = 2|\omega v_{rel}| \sin\alpha = 10\pi \frac{cm}{sec^2}$. The projection of $\boldsymbol{v}_{rel}$ lies along *MD*. Turning that projection through a right angle in the direction of the rotation of transport, i.e., counterclockwise, we obtain the direction of $\boldsymbol{a}_{cor}$ (which in the present case coincides with the direction of $\boldsymbol{a}_{tr}^{\tau}$).

The absolute acceleration of the particle M is

$$\boldsymbol{a}_a = \boldsymbol{a}_{rel} + \boldsymbol{a}_{tr}^{\tau} + \boldsymbol{a}_{tr}^{n} + \boldsymbol{a}_{cor}.$$

In order to determine the value of a_a, draw a set of axes *Oxyz* and calculate the projections of all the vectors on them. We obtain



$$a_{ax} = a_{cor} + |a_{rel}^{\tau}| = 10\pi + 10\sqrt{3} \approx 48.7\ cm/sec^2,$$

$$a_{ay} = a_{rel}\sin\alpha - a_{tr}^{n} = \frac{5\pi^2}{18}\sqrt{3} - 10\sqrt{3} \approx -12.6\frac{cm}{sec^2}.$$

$$a_{az} = -a_{rel}\cos\alpha = -\frac{5}{18}\pi^2 \approx 2.7\frac{cm}{sec^2},$$

And finally $a_a = \sqrt{a_{ax}^2 + a_{ay}^2 + a_{az}^2} = 50.4\ cm/sec^2$.

Vector $\boldsymbol{a}_a$ can be constructed according to its rectangular components along the coordinate axes *Oxyz*.